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TOWARDS A PERFORMANCE-BASED DESIGN OF PILED FOUNDATIONS: A NOVEL ANALYTICAL FRAMEWORK FOR PREDICTING PILE RESPONSE IN SAND

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ABSTRACT

In this work a novel analytical framework for the prediction of cast-in-situ piles response in sand is proposed. First, the paper describes an effective formulation for the evaluation of the shaft load-settlement response accounting for the horizontal stress increase during the loading process produced by the dilatancy occurring in the shear band. Second, an analytical solution, based on the well-known similarities between the pile tip behaviour and that of an expanding spherical cavity in an elasto-plastic medium, is proposed. The proposed methods require standard input parameters, available from ordinary in situ and laboratory tests performed in current practice. The accuracy of the proposed analytical tools is verified against well-documented full scale pile load tests.

Keywords: pile foundations, performance-based design, pile settlement, sand.

INTRODUCTION

Piled foundations, particularly those supporting critical facilities such as tall buildings, bridge piers and wind turbines, must be designed to ensure a satisfactory behaviour against generalized load conditions coming from wind and/or earthquake action. In many cases the design is ruled by serviceability considerations, i.e., by some threshold values of settlements and/or rotations which must not be exceeded.

Accurate estimation of single pile response is crucial for assessing the foundation performance and thus ensuring these serviceability requirements are met. Furthermore, since piles under axial loads exhibit a punching behaviour, even for bearing capacity calculations a performance-based design approach must be adopted.

It follows that the assessment of the single pile load-settlement response under axial load should be required for any type of design check.

Thus, with the aim of providing physically based models to be directly applied in practice, this work presents analytical solutions for the prediction of the load-settlement response of the pile shaft and the pile base, limited to bored piles in sandy soils. Each solution requires standard input parameters, given the ordinary in situ and laboratory tests employed in the current practice.

PILE SHAFT RESPONSE

It has been observed that the shaft load-settlement response can be effectively described through a hyperbolic law (Fleming, 1992), requiring only knowledge of the initial stiffness k_s and the ultimate resistance $Q_{s,lim}$.

$$Q_s = \frac{w}{\frac{1}{k_s} + \frac{w}{Q_{s,\lim}}} \tag{1}$$

where, Q_s is the shaft load and w is the pile head settlement. Alternatively, several equations for the description of the shaft load-settlement response can be employed (Bateman et al., 2022).

Different approaches can be adopted to evaluate the initial shaft stiffness k_s. In general terms, the available methods can be classified in two categories: continuum models and independent springs models. The springs-based approach has the advantages that although requires smaller computational effort than rigorous continuum-based formulations, yields predictions that are in satisfactory agreement with more complex solutions. Among these, the original solution proposed by Randolph & Wroth (1978) is perhaps the most widespread given its effectiveness despite the simple mathematical formulation. According to the authors, the initial shaft stiffness k_s can be estimated trough the following expression:

$$k_s = \frac{2\pi L G_{0,L/2}}{\ln\left[\frac{2XL(1-v_s)}{d}\right]} \tag{2}$$

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where, L is the pile length, $G_{0,L/2}$ is the soil shear modulus (at low strains) at the depth L/2, d is the pile diameter, ν_s is the Poisson's ratio of the soil and X is a dimensionless coefficient taking into account the variation of soil modulus with depth and the soil stiffness ratio at pile base (for end-bearing piles). The coefficient X is equal to 2.5 for a homogeneous half-space and to 1 for a Gibson soil on rigid bedrock at depth 2.5L (Mylonakis & Gazetas, 1998).

As regards the shaft resistance $Q_{s,lim}$, it is equal to the integral over the pile shaft of the ultimate shaft friction q_s, defined as:

$$q_s = \left(\sigma'_{h0} + \Delta\sigma'_h\right)\tan\varphi_{cv} \tag{3}$$

where σ'_{h0} is the initial effective horizontal stress, $\Delta\sigma'_{h0}$ is the effective horizontal stress increment and φ_{cv} is the soil critical state friction angle.

The horizontal stress increment $\Delta\sigma'_{h0}$, is due only to the loading process, under the hypothesis of negligible influence of the installation process and of the Poisson's effect. Indeed, at high load levels, deformations tend to concentrate within a very thin shear band around the pile shaft. Thus, deviatoric and volumetric plastic strains develop in the shear band, either dilation or contraction, depending on the state of the soil. Except for very loose sands and very high stress levels, dilatancy takes place in most soil conditions and accordingly radial displacements u directed toward the surrounding soil arise. The shear band expansion results in the horizontal stress increment $\Delta\sigma'_{h0}$ exerted by the adjacent soil. This contribution can be analytically computed adopting the following hypothesis: (a) the shear band behaves as an expansive cylindrical cavity, (b) the surrounding soil behaves as a linear elastic medium and (c) the shear band thickness is negligible compared to the pile diameter; therefore:

$$\Delta\sigma'_{h0} \cong \left[\chi \frac{\tan\psi_p w_{cv}}{2}\frac{4G}{d}\right] \tag{4}$$

where χ is a coefficient lower than unity, for practical purposes an effective value of χ = 0.6 can be adopted (Mascarucci et al., 2016); ψ_p is the angle of dilatancy at peak, it can be computed through the equation proposed by Bolton (1986); w_{cv} is the displacements required to achieve critical state conditions and G is the soil shear modulus. Note that the term $(\chi \tan\psi_p\, w_{cv})/2$ in the equation above represents the radial displacement u within the shear band.

Following the findings of Yoshida & Tatsuoka (1997) and Mascarucci et al. (2016), it can be observed that w_{cv} can be estimated through the following expression:

$$\frac{w_{cv}}{D_{50}} = 6\left(\frac{D_{50}}{1\text{mm}}\right)^{\left[0.04\left(\frac{D_{50}}{1\text{mm}}\right)-0.35\right]} \tag{5}$$

where, D_{50} is the mean particle size.

In order to account the non-linear behaviour of the soil, Mascarucci et al. (2016) suggest employing a secant (effective) stiffness in eqs. 4. Cesaro (2024) employed Yu & Houlsby (1991) solution carried out a wide parametric study and for each analysed case the secant stiffness was computed against the normalized cavity expansion. The author proposed the following empirical expression based on the results of the parametric study:

$$\begin{cases} \frac{G}{G_0} = 1 \quad \text{if} \quad u \le u_y \\ \frac{G}{G_0} = \left(\frac{u}{u_y}\right)^{-0.425} \quad \text{if} \quad u > u_y \end{cases} \tag{6}$$

where, u_y is the yielding cavity expansion; for cohesionless soil, it is equal to:

$$\frac{u_y}{d} = \frac{p'_0 \,\text{sen}\,\varphi}{4G_0} \tag{7}$$

where, p'_0 is the mean effective stress, G_0 is the soil shear modulus at law strains and φ is the soil friction angle. Note that, in Yu & Houlsby's solution the friction angle is assumed to be constant. Although this is a simplified approximation, it can be observed that adopting an average value between the initial state ($\varphi = \varphi_p$) and ultimate state ($\varphi = \varphi_{cv}$) leads to realistic results.

The effectiveness of the proposed model is checked by Cesaro (2024) against the results of full-scale load tests from well documented case histories. Figure 1 shows the theoretical and experimental results for the case history presented by Viggiani & Vinale (1983). The authors reported full scale pile load tests results performed for the design of the foundation of the Naples Law Courts' Building. The foundation consists of bored piles with a length of 42 m and diameters ranging between 1.5 and 2.2 m. In this work the results for two piles are shown, having respectively d = 1.5 m (pile A) and d = 2 m (pile B). For the fundamental properties of the soil refer to the original work.

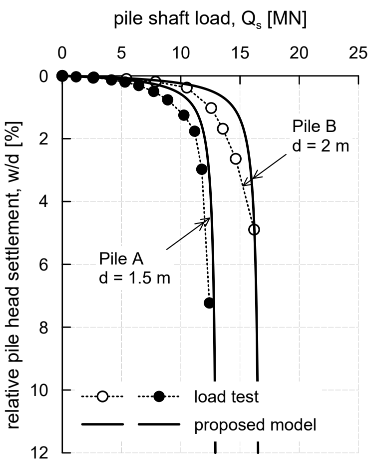

Figure 1 *Comparison with experimental data*

PILE BASE RESPONSE

The base resistance of piles in sand is generally assessed by theoretical approaches based on the limit equilibrium method and several expressions for the well-known bearing capacity factor are available from literature. However, contrary to the assumption made by such methods, experimental evidence shows that distinct failure surfaces are not observed beneath piles in sand (Vesic, 1977). Indeed, there is no visible collapse of the foundation and no clearly defined peak load since the actual failure mechanism is a punching process. In this regard, the first contribution worth of note is the solution proposed by Vesic (1977) based on the well-known similarities between the punching mechanism at the pile tip and the expansion of a spherical cavity in an elasto-plastic medium.

This work presents an analytical model for predicting the pile base load-settlement curve, extending the work of Vesic and other authors. To use the cavity expansion theory to predict the pile base load-settlement behaviour, (a) analytical solutions to compute the pressure-expansion behaviour of the cavity are needed and (b) semi-analytical correlations are required to relate the end bearing resistance to the cavity pressure and the pile base settlement to the cavity expansion. For the step (a) in this work the solution proposed by Yu & Houlsby (1991) is employed. Thus, the properties of the soil are defined by the Young's modulus E, Poisson's ratio ν, cohesion c, angle of friction φ and angle of dilation ψ. Since the response prediction of cast-in-situ piles in sandy soils is the focus of this work, the cohesion is always assumed as equal to zero. Due to space limitations, the mathematical formulation of the above solution is not shown herein.

For the step (b) the correspondence between the pile base resistance and the cavity pressure is obtained assuming, as proposed by Vesic (1977) that during the penetration of the pile a rigid cone of soil is formed at the pile base, the vertical section of the cone is a triangle with a base angle α equal to 45°+φ/2, and that the normal pressure acting on the lateral surface of the cone is equal to the cavity pressure (Fig. 2). It follows that the relationship between the pile base normal stress q_b and the cavity pressure p_{cavity} is given by:

$$q_b = p_{\text{cavity}} \frac{1}{1-\sin\varphi} \tag{8}$$

Furthermore, some simplified hypotheses are adopted in this work:

a. the initial radius of the cavity is equal to that of the pile

b. the pile tip settlement w_b is equal to the increase in the cavity radius

c. the elastic stiffness of the pile base is equal to that of a rigid disk on an elastic half space (Randolph & Wroth, 1978).

Given the hypotheses (a) and (b), the condition (c) is guaranteed by simply modifying the shear modulus, to get an equivalent G_{cavity} satisfying the equation:

$$\frac{G_{\text{cavity}}}{G_0} = \frac{(1-\sin\varphi)}{\pi(1-v)} \tag{9}$$

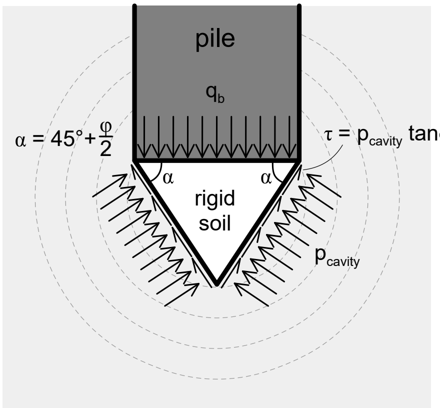

Figure 2 *Adopted conceptual model*

The reliability of this method was checked against 50 well-documented in-situ pile load tests performed worldwide, on cast in situ piles (bored, Continuous Flight Auger and Full Displacement Pile) in sandy soils by Cesaro et al. (2023) and Cesaro (2024). For each load test the load-settlement curve at pile base is available and the subsoil condition are well known. The pile load test database involves several piles geometries (diameters from 0.4 m to 2 m and lengths from 6 m to 91 m). Cesaro et al. (2023) reported for each case history the reference, pile type, pile geometry and soil properties. In Figure 3 the good performance of the proposed method is shown for 2 case histories involving bored piles.

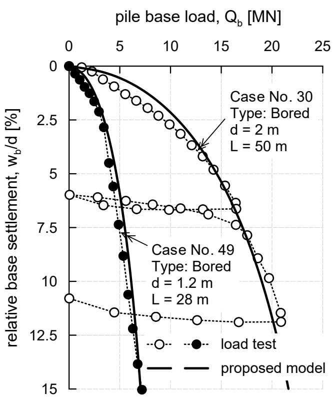

Figure 3 *Comparison with experimental data*

CONCLUSIONS

In this work analytical solutions have been presented for the prediction of the load-settlement curve of the pile shaft and the pile base, for bored piles in sandy soils. The simplest method to obtain the total load-settlement curve for an axially loaded pile, employing the solutions presented, is to adopt the hypothesis of rigid pile ($w = w_b$). However this implies the overestimation of the initial pile stiffness, because pile shortening is neglected. Alternatively it is possible to adopt the simplified method proposed by Fleming (1992) to take into account the pile compressibility. The author proposes to assume the elastic shortening of the pile as an additional settlement to that calculated with the rigid pile hypothesis. It can be assumed that the shortening occurs in two different stages: (a) the first one represents the elastic shortening which takes place during the load increase up to the complete mobilization of shaft friction and it can be computed as the shortening of the concrete column having length equal to the distance from the pile head to the centroid of the friction load transfer diagram; (b) the second stage represents the elastic shortening of the entire column occurring when the applied load exceeds the ultimate shaft load.

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