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STUDY OF NATURAL SOIL ANISOTROPY USING HOLLOW CYLINDER TESTS

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ABSTRACT

Predicting soil response to tunnelling using a TBM remains challenging due to multiple influencing factors, with the choice of the constitutive model playing a critical role. While natural soils often exhibit anisotropic behaviour, most models are based on isotropic constitutive models due to the experimental complexity of characterising anisotropy. However, incorporating anisotropy can significantly enhance the accuracy of ground displacement predictions. This study demonstrates that the Hollow Cylinder Apparatus (HCA) is an effective tool for investigating the elastic anisotropic parameters, defining a transversely isotropic soil model. The proposed approach requires a single specimen and does not need horizontal, vertical and/or inclined tests, which are not easy to perform experimentally. An analytical study of the HCA test is presented, followed by a methodology for experimentally determining these parameters. A detailed application on a natural soil sample is provided, illustrating the possibilities of the proposed approach.

Keywords: hollow cylinder apparatus, natural soil anisotropy, urban tunnel, stress rotation, laboratory test.

INTRODUCTION

Most natural soils are formed through sedimentation and develop horizontal layering at both microscopic and macroscopic scales. This suggests that they exhibit a form of transverse isotropy, where the vertical axis serves as an axis of radial symmetry. As a result, soils are often assumed to have equal stiffness in all horizontal directions and to exhibit a different stiffness in the vertical direction (Oda, 1972). It has been demonstrated that accounting for elastic stiffness anisotropy is crucial for achieving accurate finite element analysis, particularly for ground deformations above tunnels (e.g., Lee and Rowe, 1989; Gilleron, 2016). A linear transversely isotropic model can be described by five parameters, namely the vertical and horizontal Young's moduli E_v and E_h, the Poisson's ratio between the vertical and horizontal directions ν_{vh} , the Poisson's ratio in the horizontal plan ν_h , and the shear modulus between the vertical and horizontal directions G_{vh}. The shear modulus in the horizontal plan G_h is not an independent parameter but, it is dependent to E_h and ν_h. Determining the anisotropic parameters presents some difficulties. Using the classic triaxial test, one must test at least one vertical, one horizontal, and one inclined sample, which presents technical complications. Researchers also combine the triaxial test with bender element tests using wave propagation methods to examine these parameters; however, differences in the strain levels between tests can lead to some inconsistencies between the determined parameters (Reiffsteck, 2002).

The hollow cylinder test has the particularity of testing the soil under both compression and rotation (Reiffsteck and Nasreddine, 2002). It can reproduce more complex stress states than triaxial tests, particularly shear stresses between horizontal and vertical directions. In such a test, the tested sample takes the form of a hollow cylinder, with an inner diameter of 60 mm, an outer diameter of 100 mm, and a height of 200 mm. It can be subjected to external and internal pressures p_e and p_i, and back pressure (BP). In addition, a torque M_t and a vertical displacement u_0 can be applied at the base of the sample as illustrated in Figure 1.

This study demonstrates that the Hollow Cylinder Apparatus (HCA) is an effective tool for investigating the elastic anisotropic parameters. An analytical study of the HCA test is firstly presented, followed by a methodology for experimentally determining these parameters. Finally, an application on a natural soil sample is provided, illustrating the possibilities of the proposed approach.

ANALYTICAL ANALYSIS

In this study, the sign convention of continuum mechanics is adopted: stresses are positive in tension and negative in compression; deformations are positive for elongation and negative for shortening. We consider a hollow cylinder with

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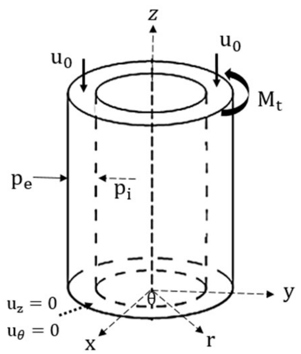

Figure 1 *Geometrical configuration and experimental conditions of the hollow cylinder test*

inner and outer radii R_i and R_e and height H. We use cylindrical coordinates (r,θ,z), and neglect the weight of the specimen. As a first approximation, the radial displacement is assumed, in this stage, to be independent of the vertical position z, while the vertical displacement is considered uniform across a section of the sample. The vertical motion and rotation of the lower base are assumed restricted.

The vertical displacement along the specimen is then:

$$u_z = \frac{u_0}{H} z \tag{1}$$

Combining the equilibrium equation and elastic Hooke's law, the following differential equation is obtained for the radial displacement:

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(ru_r)\right) = 0 \tag{2}$$

Which leads to:

$$u_r = \frac{A}{r} + Br \tag{3}$$

With A and B as constants, that can be calculated using the boundary conditions related to the radial stress at the inner and outer faces of the specimen:

$$A = -\frac{1}{2G_h}\frac{R_e^2 R_i^2}{R_e^2 - R_i^2}(p_e - p_i) \tag{4}$$

$$B = -\left(\frac{1}{2K}\frac{R_e^2 p_e - R_i^2 p_i}{R_e^2 - R_i^2} + v_{vh}\frac{u_0}{H}\right) \tag{5}$$

Where we noted:

$$K = \frac{1}{2}\frac{E_h}{(1 - v_h) - 2\frac{E_h}{E_v}v_{vh}^2} \tag{6}$$

Ilt can be observed that the horizontal shear modulus G_h is associated to the pressure difference pe-pi, and the inverse of the radial distancer. This means that the five elastic anisotropic parameters cannot all be determined without independently controlling the external pressure and the internal pressure, nor by using a conventional triaxial test where the sample is a solid cylinder. Considering the equilibrium of moments makes it possible to show that the orthoradial displacement is given by:

$$u_\theta = \frac{M_t}{G_{vh} I} zr \tag{7}$$

Where I is the polar moment of the section. The HCA rather measures the rotation angle θ:

$$\theta = \frac{M_t}{G_{vh} I} Z \tag{8}$$

Knowing the displacements, one can calculate the strain tensor then the stress tensor. One gets

$$\sigma_r = \frac{R_e^2 R_i^2}{R_e^2 - R_i^2}\left[\left(\frac{1}{R_e^2} - \frac{1}{r^2}\right)p_i - \left(\frac{1}{R_i^2} - \frac{1}{r^2}\right)p_e\right] \tag{9}$$

$$\sigma_\theta = \frac{R_e^2 R_i^2}{R_e^2 - R_i^2}\left[\left(\frac{1}{R_e^2} + \frac{1}{r^2}\right)p_i - \left(\frac{1}{R_i^2} + \frac{1}{r^2}\right)p_e\right] \tag{10}$$

$$\sigma_z = -2v_{vh}\frac{R_e^2 p_e - R_i^2 p_i}{R_e^2 - R_i^2} + \frac{E_v u_0}{H} \tag{11}$$

$$\tau_{\theta z}(r) = \frac{M_t}{I} r \tag{12}$$

The analytical study allowed us to establish explicit relationships between the applied conditions that we can control and the measurable quantities in laboratory, allowing the determination of the soil's elastic anisotropic properties through various methodologies. In the next section, we present a methodology to determine the five elastic anisotropic parameters.

IDENTIFICATION OF DRAINED SOIL PARAMETERS USING THE HCA TEST

The HCA test starts with saturation stage, by injecting water under high back pressure while a confining pressure is applied around it. The next stage is the consolidation phase, where the back pressure is kept constant, and the confining pressure is gradually increased to a value corresponding to the effective stress considered (250 kPa in this study). Consolidation is considered complete when the sample's volume stabilizes.

To determine the horizontal shear modulus G_h and the parameter K, containing four of the parameters sought, the variations of the mean radius R_m can be used. Blocking the vertical displacement, this variation can be written taking $r = R_m = \frac{R_e + R_i}{2}$ in the equation 4:

$$\frac{\Delta R_m}{R_m} = -\frac{p_e}{K_e} + \frac{p_i}{K_i} \tag{13}$$

Where we denoted:

$$\frac{1}{K_e} = \frac{1}{G_h}\frac{2}{1-\alpha^2}\left(\frac{\alpha}{1+\alpha}\right)^2 + \frac{1}{2K}\frac{1}{1-\alpha^2} \tag{14}$$

$$\frac{1}{\mathrm{K_i}} = \frac{1}{\mathrm{G_h}}\frac{2}{1-\alpha^2}\left(\frac{\alpha}{1+\alpha}\right)^2 + \frac{1}{2\mathrm{K}}\frac{\alpha^2}{1-\alpha^2} \tag{15}$$

With $\alpha = \frac{R_I}{R_e}$. The mean radial variation can be expressed in function of the variations of the inner and the outer radii as follow:

$$\frac{\Delta \mathrm{R_m}}{\mathrm{R_m}} = \frac{1}{1+\alpha}\frac{\Delta \mathrm{R_e}}{\mathrm{R_e}} + \frac{\alpha}{1+\alpha}\frac{\Delta \mathrm{R_i}}{\mathrm{R_i}} \tag{16}$$

Technically, it is difficult to measure the variations of the inner radius; so we propose a simplified method based on the variations of the sample volume Vs and the hollow volume V_h measured during the test. While the vertical displacement is fixed to zero, it can be assumed that the radial displacement takes a parabolic form during loading, being maximum at mid-height and equal to zero on the lower and upper faces. We obtain the following expressions:

$$\frac{\Delta \mathrm{R_i}}{\mathrm{R_i}} = \frac{5}{4}\left(\sqrt{1+\frac{6}{5}\frac{\Delta \mathrm{V_h}}{\mathrm{V_{h0}}}} - 1\right) \tag{17}$$

$$\frac{\Delta \mathrm{R_e}}{\mathrm{R_e}} = \frac{5}{4}\left(\sqrt{1+\frac{6}{5}\left(\alpha^2\frac{\Delta \mathrm{V_h}}{\mathrm{V_{ho}}} + (1-\alpha^2)\frac{\Delta \mathrm{V_g}}{\mathrm{V_{go}}}\right)} - 1\right) \tag{18}$$

Where V_{h0} and V_{s0} are the initial hollow and sample's volumes respectively. By varying p_e and p_i independently and measuring the mean radius variations, one can evaluate K_e and K_i. Then, one can obtain G_h and K as follows:

$$G_h = 2\left(\frac{\alpha}{1+\alpha}\right)^2\frac{K_e K_i}{K_e - \alpha^2 K_i} \tag{19}$$

$$\mathrm{K} = \frac{1}{2}\frac{\mathrm{K_i K_e}}{\mathrm{K_i - K_e}} \tag{20}$$

To determine the vertical shear modulus G_{vh}, a rotation θ must be applied progressively. In this stage, the deviatoric stress must be maintained equal to zero and p_e equals to p_i. The generated torque M_t is measured. The parameter G_{vh} can be calculated using equation 7 replacing z by the height of the sample H. The vertical modulus E_v and vertical Poisson's ratio ν_{vh} are determined by performing a vertical loading. They are equal to the initial slope of variation of the deviatoric stress and volumetric strain respectively versus the vertical strain:

$$\mathrm{q} = \mathrm{E_v}\varepsilon_\mathrm{z} \tag{21}$$

$$\varepsilon_\mathrm{v} = -(1-2\upsilon_\mathrm{vh})\varepsilon_\mathrm{z} \tag{22}$$

Finally, based on the parameters determined, the horizontal modulus E_h and Poisson's ratio ν_h can be calculated as follow:

$$\upsilon_\mathrm{h} = \frac{1-2\mathrm{G_h}\left(\frac{1}{2\mathrm{K}} + \frac{2\upsilon_\mathrm{vh}^2}{\mathrm{E_V}}\right)}{1+2\mathrm{G_h}\left(\frac{1}{2\mathrm{K}} + \frac{2\upsilon_\mathrm{vh}^2}{\mathrm{E_V}}\right)} \tag{23}$$

$$\mathrm{E_h} = 2(1+\upsilon_\mathrm{h})\mathrm{G_h} \tag{24}$$

EXPERIMENTAL RESULTS

In this section, we apply the proposed methodology to a natural soil extracted from a site in Toulouse, France, at a depth of 18 meters. It is classified as silty sand (SM) according to the Unified Soil Classification System (USCS), with an in-situ void ratio of 0.33 and a natural water content of 15%. To reproduce the relatively high natural density, we adopted compaction utilizing a full specimen press. Then, three helical shafts of different diameters are used to form manually and progressively the hole inside the sample (Figure 2). This procedure is inspired by the method described by (Reiffsteck et al., 2007).

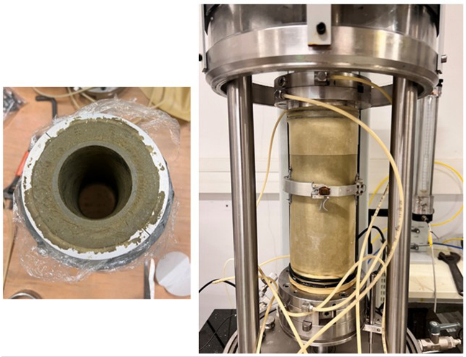

Figure 2 *Sample preparation and installation*

At two different stages, following the saturation and consolidation phases, the internal and external pressures are progressively varied around the consolidation stress at a low rate of 1 kPa/min, in order to ensure drained conditions. The variation in the mean radius of the sample, induced by the changes in internal and external pressures, is calculated as described in the previous section. This variation is used to determine the horizontal Young's modulus and Poisson's ratio and is presented in Figure 3.

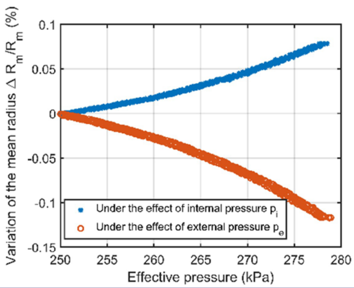

Figure 3 *The variation in the mean radius of the sample, induced by the changes in internal and external pressures*

Subsequently, a progressive torsional loading of 0.6° corresponding to a shear strain of 0.21% is applied at a rate of 0.1°/min. The variation of shear stress with respect to shear strain corresponds to the shear modulus G_{vh}. Finally, a vertical shear loading until failure is performed at a rate of 0.1 mm/min. The slope of the deviatoric stress-vertical strain curve obtained defines the vertical Young's modulus E_v. The sample's volume variation during this stage permits the determination of the vertical Poisson's ratio ν_{vh} as detailed in the previous section. The various elastic parameters are determined within a strain range of approximately 0.01%, and are summarised in Table 1.

Table 1 *Anisotropic soil parameters*

K_e	50.07 MPa
K_i	61.93 MPa
K	130.67 MPa
G_h	31.40 MPa
E_v	136.18 MPa
E_h	88.01 MPa
G_{vh}	45.15 MPa
ν_{vh}	0.45
ν_h	0.40

CONCLUSIONS AND PERSPECTIVES

This study has demonstrated the potential of the hollow cylinder test as a relevant tool for characterizing the anisotropy of the elastic behaviour of soils. The analytical approach developed led to the proposal of an original experimental program, enabling the identification of the five elastic anisotropy parameters from a single test conducted on a single specimen. The application of this protocol to a natural soil confirmed the anisotropic nature of the material, emphasizing the importance of accounting for this property in geotechnical analyses. Although often overlooked due to the experimental complexity involved, anisotropy can have a significant impact on the mechanical response of soils. As a continuation of this work, the proposed methodology will be employed to characterize a specific soil in order to calibrate a numerical simulation of tunnel excavation. This approach will make it possible to assess the actual contribution of considering anisotropy, particularly in the prediction of surface displacements induced by tunnel boring machine (TBM) advancement.

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DESIGN AND TESTING OF PILES